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Strongly-interacting Fermions from a higher-dimensional Unified Gauge Theory

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Abstract

The method of coset-space dimensional reduction is employed in order to proceed from a gauged $E_8 \times E_8'$ unified theory defined in 10 dimensions to 4 dimensions. The resulting theory comprises the Standard Model along with a strongly-interacting fermion sector which breaks the electroweak symmetry dynamically at the right scale.

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1 Introduction

High-energy experiments during the last decades have confirmed to a large extent the predictions of the Standard Model (SM). The structure of the fermion multiplets and their respective gauge interactions based on the group $SU(3)_C \times SU(2)_L \times U(1)_Y$ are described with a high precision. The nature of the Higgs sector however, which is responsible for the breaking of the $SU(2)_L \times U(1)_Y$ symmetry to $U(1)_{\rm em}$ and for the fermion masses, remains still unknown.

Efforts to describe the theory hidden behind not only the Higgs mechanism but also the fermion-generation pattern has led to various unified gauge theories. These typically contain fundamental scalar particles with masses around the weak scale and usually below 1 TeV. While a space-time global symmetry like supersymmetry can stabilize scalar masses at such low scales, it cannot explain why nature has chosen this particular value for the weak scale.

Since unified field theories use a gauge symmetry to explain the hierarchy between the strong-interactions scale and the unification scale Λ_{GUT}, it is reasonable to imagine that the hierarchy between the weak scale and Λ_{GUT} is also due to a local symmetry. This leads one to replace the perturbative Higgs sector with a non-perturbative effective one providing a dynamical symmetry breaking mechanism based on high-colour representations [1] of ordinary QCD or on new local symmetries like the ones introduced in technicolour [2] or top-colour [3] scenaria.

An effort to construct a unified theory which avoids fundamental scalars near the weak scale and in parallel addresses the strong CP problem appeared recently [4]. It introduces new fermions with interchanged weak quantum-number assignments, referred to as katoptrons, which, in contrast to the hitherto known mirror fermions, interact strongly with each other under a new "horizontal" gauge interaction.

While being unified with the other SM interactions at a unification scale Λ_{GUT} consistent not only with flavour-changing neutral-current and proton-lifetime bounds but also with small SM-neutrino masses [5], the katoptron horizontal interaction becomes naturally strong around the weak scale. Other dynamical symmetry breaking models can hardly offer such a unification perspective. This approach is also to be distinguished from models using weak horizontal interactions between SM fermions in attempts to understand family mixing and weak CP violation [6], since only the katoptrons transform under the new generation symmetry.

The resulting katoptron condensates at the weak scale play a role similar to the ordinary Higgs sector with respect to mass generation, but with a symmetry-breaking scale that is no longer arbitrary but determined by gauge-coupling unification. This constitutes an attractive solution to the hierarchy problem, since the weak scale can be expressed in terms of other known physical parameters. On the other hand, generating mass for the SM fermions depends on the breaking of this new gauge symmetry, which is denoted here by $SU(3)'$ (and introduced under the notation $SU(3)_{2G}$ in [5]). While fermion composite operators which can break $SU(3)'$ dynamically exist in the theory [4], it has still to be shown that they assume the values needed to reproduce correctly the SM fermion-mass spectrum.

The approach advocated here has some clear phenomenological advantages over previous dynamical symmetry breaking models. Not only does it produce contributions to the S and T parameters that are naturally compatible with experiment, but it is also consistent with the large deviation of the experimentally measured weak right-handed bottom-quark coupling g_R^b from its SM value [7]. This is in contrast with technicolour theories for instance, which can hardly provide a cancellation mechanism for the S parameter, the T parameter is typically large, and there are no sizable contributions to g_R^b.

The purpose of the present study is to show explicitly how such a model could result from a more fundamental theory. In particular, the 10-dimensional $E_8 \times E_8'$ model is one of the very few to possess the advantage of anomaly freedom [8] and is extensively used in efforts to describe quantum gravity along with the observed low-energy interactions in the heterotic-string framework [9]. It will be therefore used in the following as a starting point for our investigation. The results obtained indicate that it constitutes a very solid basis indeed for the understanding of our world.

2 Going from 10 to 4 dimensions

2.1 The use of coset spaces

As promised above, we start with the gauge group $G = E_8 \times E_8'$ defined in 10 space-time dimensions. The exceptional Lie group E_8 has the unique property of having its 248-dimensional fundamental and adjoint representations identical. Therefore, spin-1/2 and spin-1 particles are both taken to transform like $(\mathbf{1}, \mathbf{248})$ and $(\mathbf{248}, \mathbf{1})$ under the above group structure, and the theory is at this level supersymmetric and anomaly-free. The spin-1/2 fields are taken to be Weyl-Majorana fermions. By use of the 10-dimensional chirality operator, the $(\mathbf{1}, \mathbf{248})$ and $(\mathbf{248}, \mathbf{1})$ fermion representations are defined to be left- and right-handed respectively.

Making connection with our 4-dimensional world leads one to consider 10-dimensional space-times of the form $M^4 \times B$, where M^4 is the usual Minkowski space and B is a 6-dimensional compactified manifold the structure of which has to be determined. Coset-space dimensional reduction (CSDR) of higher-dimensional gauge theories [10]-[11] provides a very elegant method of analyzing in detail the resulting 4-dimensional models.

In the CSDR approach, the manifold B is taken to be a coset space S/R, where S and R are compact Lie groups and

$$\dim(B) = \dim(S) - \dim(R). \tag{1}$$

The group S can be considered as an R-bundle over B. The group R is taken to be a subgroup of both S and G. The fact that R is not trivial is a necessary condition for a non-trivial topology for the manifold B, something which is needed for the survival of chiral fermion fields in 4 dimensions. The fact that the 4-dimensional Lagrangian is independent of the extra coordinates is then guaranteed by gauge invariance.

Embedding the symmetry R in G gives an interesting geometrical content to some of the gauge symmetries of the theory. Dimensional reduction from 10 to 4 dimensions is thus accompanied with rank reduction resulting to a surviving gauge symmetry $H \subset G$, which is the centralizer $C_G(R)$ of the group R in G. Furthermore, the extra compactified dimensions used in CSDR offer a natural framework for the unification of gauge and scalar fields. The latter have interaction potentials which can lead to spontaneous symmetry breaking, leading us from a unified gauge theory to the SM.

Recently, higher-dimensional theories were considered beyond the classical level [12] and were given a quantum meaning in the sense of the Wilson renormalization group in agreement with the treatment involving massive Kaluza-Klein excitations [13]. The CSDR approach can therefore be exploited in the study of higher-dimensional unified quantum field theories independently of more general frameworks like string theory.

Very strict rules [10], [14], [15] determine which fields finally survive, i.e. remain massless, after this process, since gauge transformations have to be compensated by the action of the symmetry group S. These rules are a guiding light for model-building, ruling out groups that lead to unacceptable phenomenologies. It will be seen for instance that katoptrons surviving at low energies can be obtained only by coset spaces which are non-symmetric [16]. In connection with initial E_8 groups, these lead interestingly enough to an E_6 unification group.

In particular, one has to decompose the adjoint representations of the groups G and S under $R \times H$ and R respectively according to

$$\begin{aligned} \mathrm{adj}(G) &= (\mathrm{adj}(R), 1) + (1, \mathrm{adj}(H)) + \sum_i (r_i, h_i) \\ \mathrm{adj}(S) &= \mathrm{adj}(R) + \sum_i s_i. \end{aligned} \tag{2}$$

The only spin-1 fields surviving are the ones transforming under the adjoint representation of H. The spin-0 fields that appear after dimensional reduction, even though initially absent, are the ones transforming like h_i under H, and only for those i's for which $r_i = s_i$.

As regards spin-1/2 fields, one decomposes the fermion representation F of G and the spinor representation of $SO(6)$ under $R \times H$ and R respectively according to

$$
\begin{aligned}
F &= \sum_i (r_i, h_i) \\
\sigma &= \sum_i \sigma_i .
\end{aligned} \tag{3}
$$

The only fermion fields surviving are the ones transforming like h_i under H, and only for those i's for which $r_i = \sigma_i$.

When studying $E_8 \times E_8'$ models [17], it is customary to identify the origin of the fields transforming under E_8' with some obscure "hidden" or "shadow" world that interacts only gravitationally with ours. The philosophy here is different, because katoptron fermions originate from this new world. Since these fermions will finally assume the role of a dynamical Higgs sector, they should have quantum-number assignments similar (but not identical) to the ones of their SM partners.

A way to achieve this goal is to make use of a discrete abelian subgroup of G consisting of two elements, which we denote by $Z_2^{E_6}$. The action of its non-trivial group element corresponds to an outer automorphism that interchanges the E_6 subgroups of the two E_8's. (Analogously, $Z_2^{E_8}$ interchanges the two E_8's.) Making use of $Z_2^{E_6}$ has the effect of reducing further the rank of the surviving symmetry H in a manner analogous to the construction in Ref.[18], as will be seen shortly. In the following, a particular 6-dimensional non-symmetric coset space is analyzed and shown to lead to an acceptable phenomenology.

2.2 CSDR with $S = Sp(4)$, $R = (SU(2) \times U(1))_{\mathrm{non-max}}$

We consider a Lie group $R = SU(2) \times U(1)$ embedded non-maximally into $S = Sp(4)$ and into $E_8 \subset G$, i.e. into the exceptional group under which the Weyl-Majorana fermions of the model are left-handed. The Euler characteristic of $Sp(4)/(SU(2) \times U(1))_{\rm non-max}$ is equal to $\chi = 4$, and a priori the number of copies of the fermion representations is, according to the index theorem, equal to $|\chi/2| = 2$.

Compactifying on $B_0 = S/R$ leads to the following decompositions of the adjoint and spinor representations of $SO(6)$ and $Sp(4)$ under R respectively:

$$\begin{array}{ll} SO(6) \supset (SU(2) \times U(1))_{\rm non-max}, & \mathbf{4} = (\mathbf{1}, 0) + (\mathbf{1}, 2) + (\mathbf{2}, -1) \\ & \bar{\mathbf{4}} = (\mathbf{1}, 0) + (\mathbf{1}, -2) + (\mathbf{2}, 1) \\ Sp(4) \supset (SU(2) \times U(1))_{\rm non-max}, & \mathbf{10} = (\mathbf{1} + \mathbf{3}, 0) + (\mathbf{2}, \pm 1) + (\mathbf{1}, \pm 2) \end{array} \tag{4}$$

The adjoint representation of E_8 decomposes under $(SU(2) \times U(1))_{\rm non-max} \times E_6$ as follows:

$$\begin{aligned} \mathbf{248} &= (\mathbf{1}, 0, \mathbf{78}) + (\mathbf{1} + \mathbf{3}, 0, \mathbf{1}) + (\mathbf{2}, \pm 3, \mathbf{1}) \\ &+ (\mathbf{1}, -2, \mathbf{27}) + (\mathbf{2}, 1, \mathbf{27}) + (\mathbf{1}, 2, \overline{\mathbf{27}}) + (\mathbf{2}, -1, \overline{\mathbf{27}}) \end{aligned} \tag{5}$$

These decompositions are not altered if the compactification is performed on the space $B = (S/R) \times (Z_2^{E_8}/Z_2^{E_6})$, i.e. when R is replaced by $\tilde{R} \equiv R \times Z_2^{E_6}$ and the corresponding fields are taken to be $Z_2^{E_6}$ singlets, with $Z_2^{E_6}$ defined as previously. Since $C_{E_8}(R) = E_6 \ \left(\times Z_2^R \times U(1)\right)$, with $Z_2^R \times U(1)$ the center of $SU(2) \times U(1)$ (the superscripts of the various Z_2 symmetries in this paper have each obviously different meaning), and $C_{E_8'}(E_6') = SU(3)'$, the centralizer $C_G(\tilde{R})$ is equal to

$$H = E_6^D \times SU(3)' \ \left(\times Z_2^R \times U(1)\right), \tag{6}$$

where E_6^D is the diagonal subgroup of the E_6 subgroups of the two E_8's.

That H given above is indeed the surviving gauge symmetry after compactification can be checked explicitly by enumerating the spin-1 degrees of freedom which are left invariant by the action of $\tilde{R}$. In the absence of the $Z_2^{E_6}$ symmetry, H would have been given by $E_6 \times E_8' \left(\times Z_2^R \times U(1)\right)$. The role of $Z_2^{E_6}$ is to keep only the diagonal subgroup of $E_6 \times E_6' \subset E_8 \times E_8'$ unbroken, eliminating all skew-symmetric contributions. Physically, it renders the compactification process more symmetric with respect to the two E_8's.

Furthermore, the SM-fermion quantum numbers under $Z_2^R \times U(1)$, the center of $SU(2) \times U(1)$, are equal to $(1, -2), (1, 1,)$ and $(-1, 1)$. The center survives after CSDR, and it is identified in the following with the family symmetry of the SM which differentiates between the three SM generations. This symmetry is taken to be global and the $U(1)$ coupling is accordingly switched-off in order to avoid problems with flavour-changing neutral currents at lower energies.

Under the gauge structure $H = E_6^D \times SU(3)' \left(\times Z_2^R \times U(1)\right)$ defined above, the CSDR rules give the following surviving 4-dimensional fields:

$$\begin{array}{rcl} \text{spin}-1/2: & (\mathbf{27},\mathbf{3}) & \text{Katoptrons} \\ & (\mathbf{78},\mathbf{1})+(\mathbf{1},\mathbf{8}) & \text{Vector fermions} \\ & 2\times(\mathbf{27},\mathbf{1})_a & \text{Include SM fermions} \\ \text{spin}-0: & 2\times(\mathbf{27},\mathbf{1})_a & \text{Higgs sector} \end{array} \tag{7}$$

where the fields transforming under E_6^D as $\mathbf{27}$ and $\overline{\mathbf{27}}$ are identified by the Majorana condition, and the subscript $a = 1, 2, 3$ serves as a generation index corresponding to $Z_2^R \times U(1)$. In the above, we also indicate in which sector fields which are known to us from the SM and katoptron model are contained. The vector fermions transforming like $(\mathbf{1}, \mathbf{8})$ and $(\mathbf{78}, \mathbf{1})$ are not protected by any gauge symmetry, so according to the general argumentation on the survival hypothesis they acquire large gauge-invariant masses of the order of the compactification scale and disappear from the low-energy spectrum. The torsion of the non-symmetric space B [19] is taken to be such that katoptrons remain massless.

The present coset space admits two different scales [20], something that could be useful in the subsequent breaking of H, as will be discussed later. Furthermore, it can be checked that this breaking leads to an anomaly-free 4-dimensional theory, since it satisfies the equation

$$l(G) = 60, \tag{8}$$

where $l(G)$ is the sum of the indices of all the representations of R appearing when the adj(G) representation is decomposed under $R \times H$ [21]. After having analyzed the geometrical rank reduction of G down to H, one has to study the subsequent breaking of H down to the SM.

3 Symmetry breaking to the Standard Model

3.1 Breaking by Wilson lines

The simply-connected group $S = Sp(4)$ considered has a Z_2 symmetry as center (recall that $Sp(4)/Z_2 \approx SO(5)$), and this can be employed here to serve in a gauge-symmetry breaking mechanism by Wilson lines [22]. The embedding of this abelian discrete symmetry in the $SU(2)_L$ subgroup of E_6^D, which we denote by $Z_2 \equiv Z_2^{SU(2)_L}$, can be defined via the following homomorphism involving its non-trivial group-element g [23]:

$$Z_2^{SU(2)_L} \ni g \longrightarrow U_g = \mathbf{1} \otimes \mathbf{1} \otimes \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix} \in SU(3)_C \times SU(3)_R \times SU(3)_L \quad (9)$$

where $SU(3)^{\otimes 3}$ is a maximal subgroup of E_6^D, the second and third row of the (diagonal) $SU(3)_L$ factor correspond to $SU(2)_L$, and of course $U_g^2 = 1$.

We then consider dimensional reduction over the coset space

$$\tilde{B} = \left(S/(R \times Z_2^{SU(2)_L})\right) \times (Z_2^{E_8}/Z_2^{E_6}), \quad (10)$$

The original gauge group G is broken at the compactification scale, which is identified here with the gauge-coupling unification scale Λ_{GUT}, down to

$$H' = SU(6) \times SU(2)_L \times SU(3)' \ \left(\times Z_2^R \times U(1)\right), \quad (11)$$

where $SU(6) \times SU(2)_L \subset E_6^D$. The original $|\chi/2| = 2$ copies of fermion and scalar fields are then further reduced to a single copy due to the action of $Z_2^{SU(2)_L}$. Obviously, one has to distinguish the topological role of this symmetry from the role of Z_2^R which merely differentiates the fermion families via quantum numbers.

A side effect of $Z_2^{SU(2)_L}$ is to break the original supersymmetry at the compactification scale, since fermion fields lose some of their bosonic partners. It is reminded here that the present model does not need low-energy supersymmetry, since the hierarchy problem is solved by the gauge symmetry $SU(3)'$ [5].

3.2 Further breaking by a Higgs mechanism

One of the scenarios presented in [23] is subsequently realized. The Higgs fields transform under $SU(6) \times SU(2)_L$ like $(\mathbf{6}, \mathbf{2}) + (\mathbf{15}, \mathbf{1})$. Only the $(\mathbf{15}, \mathbf{1})$ Higgses which are invariant under $Z_2^{SU(2)_L}$ remain light, and one of their copies is taken to develop a non-zero vacuum expectation value at the compactification scale. This breaks spontaneously the gauge symmetry further down to

$$H'' = SU(4)_{PS} \times SU(2)_R \times SU(2)_L \times SU(3)' \, \Big(\times Z_2^R \times U(1) \Big) \, , \tag{12}$$

where $SU(4)_{PS}$ is the usual Pati-Salam symmetry. The **27** representation of E_6 decomposes under $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ like

$$\mathbf{27} = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}, \mathbf{2}) + (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}) \tag{13}$$

The remaining generations of Higgses transform under $SU(4)_{PS} \times SU(2)_R$ like $(\mathbf{4}, \mathbf{2}) + (\bar{\mathbf{4}}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$. The $(\mathbf{4}, \mathbf{2})$ Higgs field is subsequently taken to acquire a non-zero vacuum expectation value and break spontaneously the gauge symmetry $SU(4)_{PS} \times SU(2)_R$ further down to $SU(3)_C \times U(1)_Y$ at the Pati-Salam scale Λ_{PS}, giving the final symmetry

$$H''' = SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)' \, \Big(\times Z_2^R \times U(1) \Big) \, , \tag{14}$$

which includes the familiar SM groups. The fact that the coset space considered in the last section admits two different scales could be at the origin of the relatively small hierarchy between the Pati-Salam symmetry breaking scale Λ_{PS} and the unification scale Λ_{GUT} [5].

There is no surviving symmetry preventing scalar particles from obtaining large masses after these breakings. Spin-1/2 particles remain light only if they are chiral, the others gaining compactification-scale masses. One then recovers the gauge and matter content which is the starting point of [4], by taking the fermions in the representations of the original E_8 and E_8' groups to be left-handed and right-handed respectively, as stated in the beginning.

Therefore, one reproduces the three SM generations and a katoptron generation which has interchanged left-right $SU(2)_L \times SU(2)_R$ quantum numbers and transforms in addition in the fundamental representation of the gauge group $SU(3)'$. Under H''', the fields transform like

$$
\begin{array}{rl}
\text{SM fermions} & \text{Katoptrons} \\
q_L : (\mathbf{3},\ \mathbf{2},\ 1/3,\ \mathbf{1})_a & q_R^K : (\mathbf{3},\ \mathbf{2},\ 1/3,\ \mathbf{3}) \\
l_L : (\mathbf{1},\ \mathbf{2},\ -1,\ \mathbf{1})_a & l_R^K : (\mathbf{1},\ \mathbf{2},\ -1,\ \mathbf{3}) \\
q_R^c : (\bar{\mathbf{3}},\ \mathbf{1},\ {}^{-4/3}_{+2/3},\ \mathbf{1})_a & q_L^{K\,c} : (\bar{\mathbf{3}},\ \mathbf{1},\ {}^{-4/3}_{+2/3},\ \mathbf{3}) \\
l_R^c : (\mathbf{1},\ \mathbf{1},\ {}^{0}_{2},\ \mathbf{1})_a & l_L^{K\,c} : (\mathbf{1},\ \mathbf{1},\ {}^{0}_{2},\ \mathbf{3}),
\end{array}
\tag{15}
$$

where the superscript K denotes katoptron fields, c charge conjugation, the subscripts L and R left- and right-handed fields, and q and l quark and lepton fields respectively. The group $SU(3)'$ is asymptotically free and provides the mechanism responsible for the dynamical breaking of the electroweak symmetry at the right scale via katoptron condensates.

4 Discussion

Starting with a higher-dimensional gauge field theory, we presented an effort to produce a picture consistent with current phenomenology and which in addition includes a dynamical Higgs sector. The need to obtain eventually the SM group structure at lower energies in 4 dimensions places severe constrains on the compactification manifolds considered. The gauge-symmetry-breaking sequence of [5] can be reproduced by use of Wilson lines for example, if the group manifold S has Z_2 as center, so $S = Sp(4)$ is left as a unique choice (for S semisimple) leading to a 6-dimensional non-symmetric manifold B.

Moreover, in order to make connection with the unification picture presented in [5], one has to note that the E_6 group with three generations seems to be favored over $SO(10)$ with 4 generations considered in that reference as a unification symmetry. In all other respects, the results and conclusions of [5] remain unaltered, since the scenario with 4 generations was rejected there for other reasons.

The likelihood of the scenario presented here should be tested not only for its theoretical consistency but also for its phenomenological relevance in forthcoming experiments [7]. It would then constitute one interesting example trying to connect the abstract mathematical world of the Planck scale with the experimental physical reality of collider data.

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